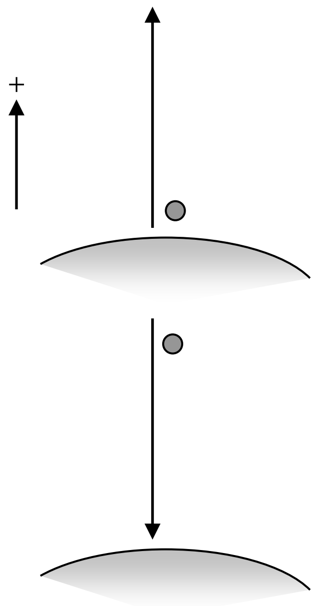
SPH4U: Rocket Science!

Or, the science of movement in a gravitational field.

Part A: Ascent, Descent

Consider a rock traveling straight up and in down in the earth's gravitational field under the influence of gravity alone near the surface of the earth. Indicate in the chart whether the quantity is increasing or decreasing in magnitude. For velocity and work indicate the sign.



	Trip Up	Trip Down
v		
E_k		
E_g		
E_T (Total Energy)		
Work by Force of Gravity		

1. When we calculate E_g, we use the value g = 9.8 N/kg. What happens to this value as we get quite far from the earth? What is the reason for this change?

2. Write down the equation for Newton's Law of Universal Gravitation. Sketch a graph of the force of gravity over great distances. Mark two separate distances, d_1 and d_2. Use the text to help.

F_g =

3. How do you graphically represent the work done on an object moved from d_1 to d_2? Show this on the graph.

Part B: Gravitational Potential Energy

1. Mathematically we describe the gravitational potential energy (GPE) at a given position when using the equation: $E_g = -\frac{GMm}{r}$ which is the result of the calculus work describing the area under the graph in Part A Q#2.

2. Fill in the chart showing the gravitational potential energies for a 1 kg rock at different positions from the centre of the earth. ($R_e = 6.4 \times 10^6$ m. $M_e = 6.0 \times 10^{24}$ kg) Sketch this as a graph.

Distance from centre of earth	E_g
R_e	
$2R_e$	
$3R_e$	
$4R_e$	
$6R_e$	
$8R_e$	
Really, really far	

Note! Really, really far means an infinitely large distance

3. Use *greater than* or *less than* symbols to compare:
   $E_g(R_e)$ $E_g(2R_e)$
   $E_g(R_e)$ E_g(really, really far)

4. According to your graph, at what position is the greatest amount of energy stored in the gravitational field?

5. What is the change in GPE when the rock travels from:
   a) $2R_e$ to $6R_e$
   b) $8R_e$ to $4R_e$
   c) R_e to infinity
   d) infinity to R_e

6. Is energy being stored in or returned from the gravitational field when ΔE_g is:
   a) positive?
   b) negative?

7. Imagine a small rocket engine is attached to the rock. How much work is done by the rocket engines to move the rock from:
   a) $2R_e$ to $6R_e$,
   b) R_e to infinity

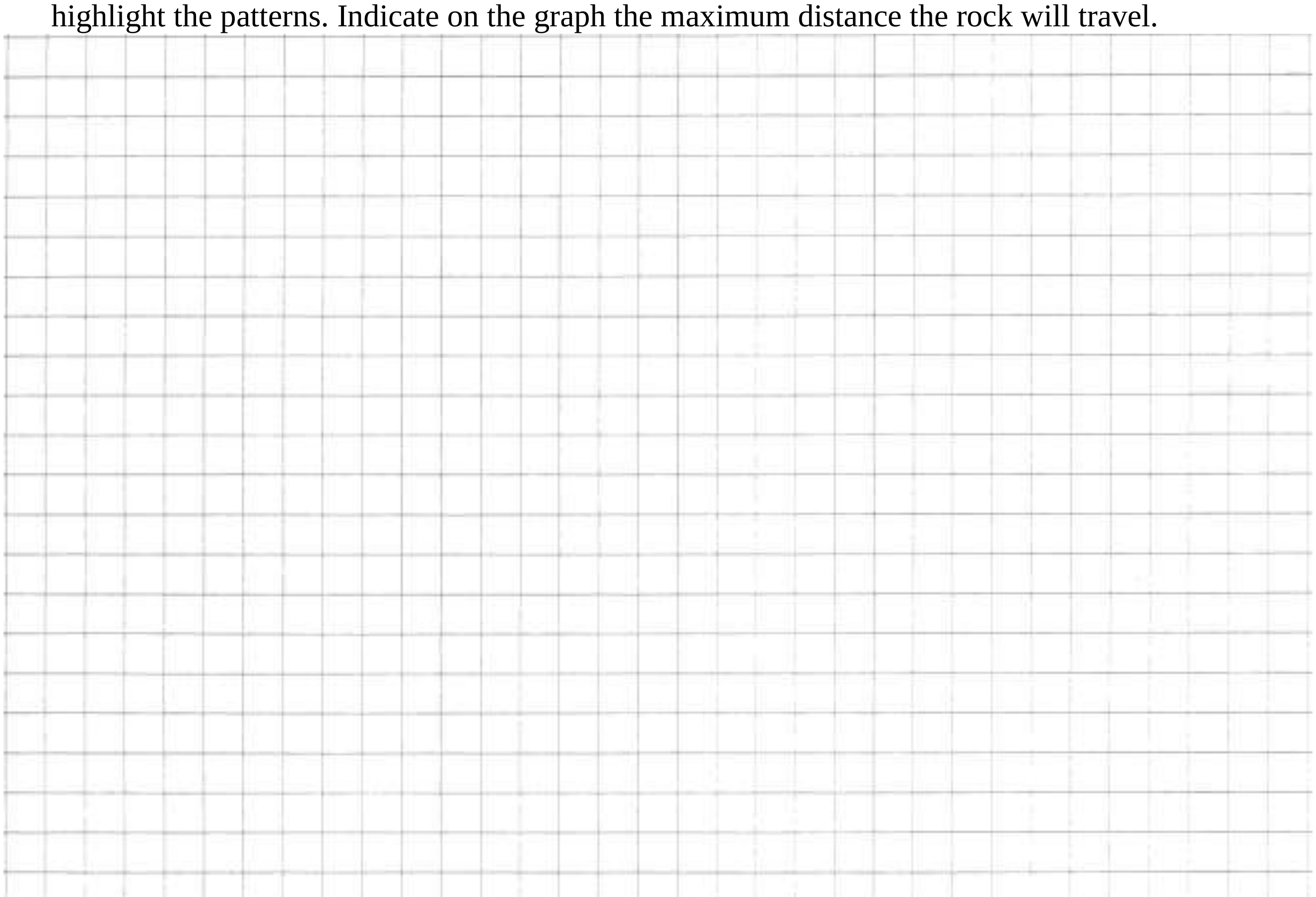
Part C: Velocity and Gravitational Fields

1. Complete the chart for the 1 kg rock and its trip through the earth's gravitational field. The rock was powerfully launched straight up from the surface of the earth at 1.0 x 10^4 m/s. **Note**: An object's **kinetic energy** must be a positive value – a negative value is impossible.

Distance from centre of earth	E_g	E_k	velocity	E_T
R_e				
$2R_e$				
$3R_e$				
$4R_e$				
$6R_e$				
$8R_e$				
Really, really far				

2. Is it possible to find the rock at a distance of $6R_e$? What eventually happened to the rock? Explain.

3. Plot on one graph E_g, E_k and E_T as a function of distance. Use smooth curves and straight lines to highlight the patterns. Indicate on the graph the maximum distance the rock will travel.



4. Explain how you can use the values of E_T and E_g on the graph to determine the rock's maximum height.

5. How much kinetic energy should the rock be launched with to reach a maximum distance of:
   i. $3R_e$
   ii. $6R_e$

6. With what velocity should the rock be launched to reach a maximum distance of $4R_e$?

7. Complete the chart again, but now the rock has an initial velocity of 1.2×10^4 m/s.

Distance from centre of earth	E_g	E_k	velocity	E_T
R_e				
$2R_e$				
$3R_e$				
$4R_e$				
$6R_e$				
$8R_e$				
Really, really far				

8. Plot a graph that shows E_g, E_k and E_T as a function of distance.

9. At what distance from the earth will the rock in this example finally come to rest?

10. The rock is launched from Earth such that it will come to rest when it has travelled an infinite distance from Earth. With what E_k was it launched?

11. Derive an algebraic expression that gives the launch velocity for any object to reach a very great distance (infinity) from the earth with essentially no (zero) kinetic energy? This is called the **escape velocity**.

12. Complete the chart for escape velocities.

Object	Radius (m)	Mass (kg)	v_{escape} (m/s)	v_{escape} (km/h)
Earth	$6.38x10^6$	$5.98x10^{24}$		
Moon	$1.74x10^6$	$7.35x10^{22}$		
Jupiter	$7.15x10^7$	$1.90x10^{27}$		

13. Imagine the earth and all its matter is compressed.
    a) What size of radius for the earth would give the maximum escape velocity (c= 3.0×10^8)?

    b) What would happen to all objects trying to escape if the earth was compressed even further? What has been created?

14. What is the total energy of an object moving (consult your charts Q#1 and 6)
    a) at its escape velocity?

    b) slower than its escape velocity?

    c) faster than its escape velocity?

Part D: Orbits

1. An object is moving in a circular orbit around the earth with some radius *R*. Use the Law of Universal Gravitation and Newton's 2^{nd} Law to determine the object's orbital velocity.

2. Write an expression for the kinetic energy of an object in a circular orbit. Write this in terms of the object's GPE.

3. Write an expression for the total energy of an object in a circular orbit. Write this in terms of the object's GPE.

4. To calculate the energy necessary to move from one situation to another we compare the total energy in each. How much work must be done by a rocket engine to transfer a 1000 kg satellite from an orbit of radius $2R_e$ to $3R_e$?

5. How much work must be done by a rocket engine to lift a 1000 kg payload from rest on the earth's surface to a circular orbit of radius $2R_e$?

6. The **binding energy** is the work that must be done to allow an object to escape to infinity. What is the binding energy for an object:
   a) on the surface of the earth

   b) in a circular orbit around the earth