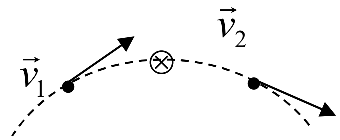
SPH4U: Radial Acceleration

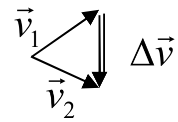
What is special about the motion of an object moving in a circle? Only vectors will tell!

A: Velocity Vector Diagrams

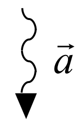
To find the radial acceleration of an object moving in circular motion, we will use a velocity vector diagram and learn a trick for choosing the two velocity vectors.



We want to estimate the acceleration for an object when it passes the middle point (⊗). Draw the initial velocity vector (v_1) a small time interval before the point. Draw the final velocity vector (v_2) an equal time interval after the point.



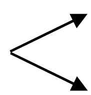
Redraw the two velocity vectors tail to tail. Draw the change in velocity vector going from the tip of v_1 to the tip of v_2.

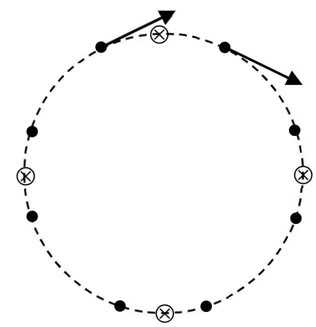


The acceleration, a, is in the same direction as the change in velocity, Δv, and has a magnitude $\Delta v/\Delta t$.

B: Acceleration and Circular Motion

1. **Represent.** An object moves at a constant speed in a circle. Use the vector subtraction technique to estimate the acceleration vector at the four moments shown in the diagram. **Neatly** show your vector work for each example (the irst example is almost complete). Draw a wiggly acceleration vector inside the circle for each moment.

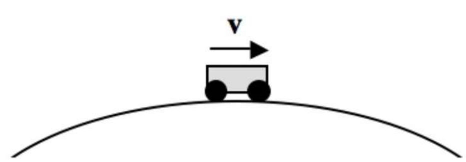




An object moving in a circle experiences an acceleration (a_r) in the radial direction pointing towards the centre of its circular path. This acceleration is called the **radial acceleration** (in the textbook this is called the centripetal acceleration).

2. **Reason.** How does the pattern for the acceleration vectors compare with the rule you developed for the net force in the yesterday's investigation? What law of physics is this a result of?

3. **Represent.** A cart glides across the top of a low-friction, circular track. We will focus on the moment in time when the cart is at the highest point. Draw a velocity vector diagram to find the acceleration at that time. Draw an ID and FD, and explain how the diagrams agree according to Newton's 2nd law. Hint: at this moment, there are no important horizontal forces.



Velocity Vector Diagram	ID	FD	Agreement?

4. **Apply.** How would it *feel* if you were a passenger on the cart in this situation? Use the FD to help explain.

C: The Pendulum of Fate

In our classroom we have a 0.50 kg mass hanging on the end of a 1.0 m string. The string is connected to a spring scale.

1. **Predict.** The mass is at rest, hanging from the scale. Predict the spring scale reading. Justify your prediction.

2. Imagine you pull the mass to the side and release it such that it swings like a pendulum. Even though the mass does not move in a complete circle, this is still circular motion!
   (a) **Represent.** Apply your new understanding of circular motion when the mass is at its lowest point. Draw a velocity vector diagram and FD for the lowest point in the swing.

   (b) **Predict.** How will the spring scale reading as the mass swings past its lowest point compare with the reading when the mass was at rest? Explain.

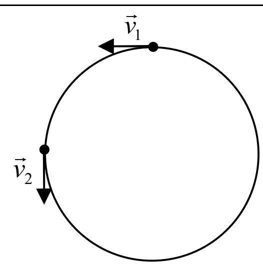
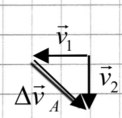
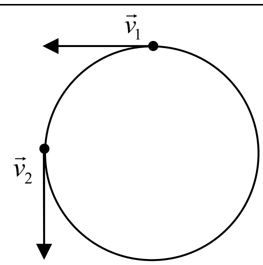
Velocity Vector Diagram	FD – Lowest Point

3. **Test.** Perform the experiment. Explain how the result agrees with our understanding of acceleration and force in circular motion.

D: Acceleration, Speed and Radius

Two identical physics buggies travel at a constant speed along two identical circular paths. Buggy A moves with a speed v and Buggy B with a speed $2v$. Buggy A completes a ¼ trip in a time Δt_A and experiences a change in velocity Δv_A.

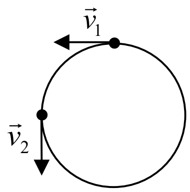
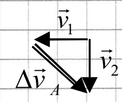
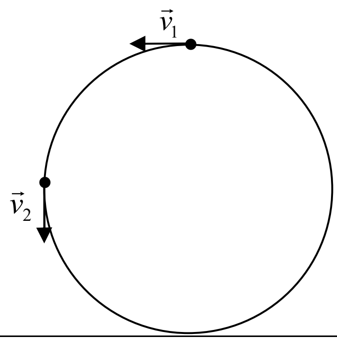
1. **Represent.** Use a velocity vector diagram to help compare the radial accelerations of the two buggies.

	Illustration	Velocity Vector Diagram	Time Interval	Acceleration
Buggy A			For a ¼ trip: Δt_A	$a_A = \frac{\Delta v_A}{\Delta t_A}$
Buggy B		How does Δv_B compare with Δv_A?	How does Δt_B compare with Δt_A?	Put it all together here: $a_B = \frac{\Delta v_B}{\Delta t_B}$ $=$

2. **Reason.** How would the acceleration change if the speed was tripled? (Be careful!)

Two identical buggies travel with the same constant speed along two circular paths with different radii. Buggy A moves in a circle of radius r and buggy B moves in a circle with radius $2r$. Buggy A completes a ¼ trip in a time Δt_A.

3. **Represent.** Use the vectors subtraction technique to compare the radial acceleration of each buggy.

	Illustration	Velocity Vector Diagram	Time Interval	Acceleration
Buggy A			For a ¼ trip: Δt_A	$a_A = \Delta v_A / \Delta t_A$
Buggy B				

4. **Reason.** How does the magnitude of the acceleration depend on the radius? (Use the word "proportional")

5. **Speculate.** We first figured out that the acceleration is proportional to the speed *squared* ($a \propto v^2$). In the second example, we saw that the acceleration is *inversely* proportional to the radius ($a \propto 1/r$). Combine these two results and create one equation for the magnitude of the radial acceleration during circular motion.

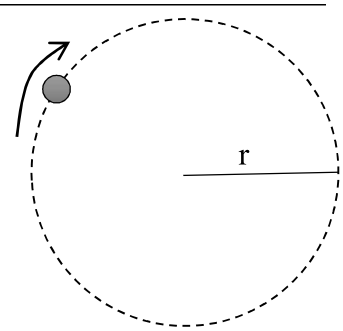
Equation for Radial Acceleration

**Check your result with your teacher **

When we analyze circular motion, it is often easiest to measure the frequency (f) or the period (T) of the circular motion rather than the speed. We can create two other handy equations for the radial acceleration using those quantities: $a_r = 4\pi^2 r f^2 = 4\pi^2 r/T^2$. Frequency is the number of rotations per second and is measured in hertz (Hz). Period is the time to complete one rotation.

F: Test the Acceleration Expression

1. **Derive.** An object is moving in a circle with radius r and period T. Start with the equation for the radial acceleration you have just developed. Use your knowledge of circles (hint: perimeter and T) to create an expression for the speed of the object. Eliminate v from your acceleration equation and create the two other equations shown above.



2. **Predict.** According to our understanding of acceleration and net force, explain what happens to the magnitude of the centripetal force if an object spins faster with the same radius.